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ESTIMATION OF A LARGE AREA

CROP ACREAGE INVENTORY USING

REMOTE SENSING TECHNOLOGY*

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0. SUMMARY

Based upon the existing remote sensing capabilities, the useful information about the acreage of some crop of economic interest can be obtained from multispectral scanner measurements acquired over an agricultural area. If the goal is to determine the acreages covered by various crops over some large area such as the continental United States, then some sampling procedure will be necessary since it would not be practical to collect and process a set of scanner data covering the entire area.

In this report we develop a model for the evaluation of acreages (proportions) of different crop-types over a geographical area using a classification approach and give methods for estimating the crop acreages. If prior information is available on the classification errors associated with the classification algorithm used, the estimation method provides the best estimate for the crop acreages. Otherwise, the method would first require a certain amount of ground truth in the area of interest to be obtained so that the classifier can be trained and the classification errors estimated.

If the main interest lies in estimating the acreages of a specific crop-type such as wheat, it is suggested to treat the problem as a two-crop problem: wheat vs. non-wheat, since this simplifies the estimation problem considerably. The error analysis and the sample size problem is investigated for the two-crop approach. Certain numerical results for sample sizes are given for a JSC-ERTS-1 data example on wheat identification performance in Hill County, Montana and Burke County, North Dakota. Lastly, for a large area crop acreages inventory we suggest a sampling scheme for acquiring sample data and discuss the problem of crop acreage estimation and the error analysis.

1. INTRODUCTION

In recent years the development of several automatic data processing techniques for statistical pattern recognition has enhanced considerably the scope of remote sensing technology for the study of earth resources, particularly in the field of agriculture. It now appears that a system for performing a large area crop inventory can be developed on the basis of existing remote sensing capabilities.

The data handling and analysis for remotely sensed agricultural resources over a large area may not be feasible both from technical and economical view-points if each scanned data point is being processed for its recognition. For example, if a complete recognition is desired for an ERTS scene, it would require processing over half a million data points. As such, an important requirement for any system to be developed for a large area crop inventory should be to have a suitable crop acreage estimation technique that uses only a sample of the unlabeled remotely sensed data obtained for the area of interest for the purpose of recognition.

In this report we discuss a large area crop acreage estimation procedure that would meet this requirement for the system. We develop a model for the evaluation of crop proportions for an agricultural area and provide methods for crop acreage estimation, taking into consideration the classification errors likely to arise in labeling remotely sensed data. The error analysis for the model is studied and expressions for variances of different estimates are given, in general as well as in specific cases. For the two-crop situation, the problem of sample size is investigated and certain numerical results for the sample size are provided. Next, we extend the scope of our study to investigate a large area crop inventory.

2. CROP PROPORTIONS MODEL

Suppose there are m different crops $\pi_1, \pi_2, \ldots, \pi_m$ in the agricultural area of interest and that every data point is identifiable with respect to one of these crops. Let p_i denote the proportion of pixels in π_i, i=1,2,...,m. Considering a random sample of n unlabeled remotely sensed data points, let n_i be the number of points classified into π_i, i=1,2,...,m, using a classification algorithm. Suppose $n(i|j)$ is the number of data points to be in π_j but classified into π_i, then

$$n_i = n(i|1) + n(i|2) + \ldots\ldots + n(i|m)$$

and

$$\frac{n_i}{n} = \sum_{j=1}^{m} \frac{n(i|j)}{n}, \qquad i=1,2,\ldots,m \tag{2.1}$$

are the observed crop proportions for the sample data under the classification algorithm used. The observed proportion n_i/n is a biased estimate of p_i since it estimates unbiasedly $E[n_i/n]$ given by

$$e_i = \sum_{j=1}^{m} E\left[\frac{n(i|j)}{n}\right]$$

$$= \sum_{j=1}^{m} p_j \, P(i|j) \tag{2.2}$$

where $P(i|j)$ denotes the probability of classifying a data point from π_j into π_i under the classification algorithm. It may be pointed out that processing of remotely sensed data for total recognition would lead to an evaluation of the expected classified crop proportions e_i's instead of the actual crop proportions p_i's. Of course, if the classification algorithm performs so well that the classifiction errors are sufficiently small, e_i will be close enough to p_i, $i=1,2,\ldots,m$. But most statistical pattern recognition techniques for processing of remotely sensed data are expected to be fallible and thereby the two types of proportions are not going to be near equal. Henceforth in our discussion we will assume that $P(i|j) > 0$ for at least one j different from i.

Denoting the observed proportion n_i/n by $\hat{e}_i$, $i=1,2,\ldots,m$, it follows from (2.2) that

$$e = E[\hat{e}]$$

or

$$e = Pp \tag{2.3}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix}, \qquad p = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{bmatrix}$$

and

$$P = \begin{bmatrix} P(1|1) & P(1|2) & \cdots & P(1|m) \\ P(2|1) & P(2|2) & \cdots & P(2|m) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ P(m|1) & P(m|2) & & P(m|m) \end{bmatrix}.$$

Accordingly, the vector of actual crop proportions

$$p = P^{-1}e \tag{2.4}$$

are obtained subject to $\sum_{i=1}^{m} p_i = 1$ provided e and P are known.

case of $\hat{\hat{p}}$ not only the estimate $\hat{\hat{p}}$ itself but bias as well as mean square error quantities will also depend upon how $\hat{P}$ is obtained. One solution for $\hat{P}$ and the probability distribution of its components is suggested therein, as well.

3. TWO-CROP APPROACH

Sometimes the main interest is in estimating the acreage of a specific crop type in the area of interest. In that case one approach to the acreage estimation problem lies in considering π_1 to be the specific crop type and π_0 to be the "other crop" consisting of the remainder of the crops, and then treating it as a two-crop situation. However, lumping of different crops together for the "other crop" would require certain caution and should be judged in terms of the classification performance for the two-crop case as against that for the case of the original set of crops. For the Gaussian maximum likelihood classifier, Basu and Odell (1973) have investigated this problem and have shown that the classification performance for the class of main interest may or may not improve when the classification is performed using the two-class approach. But the problem under this approach is greatly simplified and, barring extreme cases, perhaps it will provide satisfactory solutions in the remote sensing situation when interest only lies in ascertaining the acreage cover of one specific crop.

Now considering two crops π_1 and π_0, let $P(1|0) = \Phi_1$ and $P(0|1) = \Phi_2$ for the probabilities of misclassification when a certain classifier is used. We will assume that $\Phi_1 + \Phi_2 \neq 1$. Then

$$P = \begin{bmatrix} 1 - \Phi_2 & \Phi_1 \\ \Phi_2 & 1 - \Phi_1 \end{bmatrix} .$$

If p_1 and p_2 are the actual crop proportions of π_1 and π_0, respectively, whereas e_1 and e_2 are their respective expected classified crop proportions under the classifier used, it follows from (2.3) that

$$e_1 = (1-\Phi_2)\ p_1 + \Phi_1\ (1-p_1) \tag{3.1}$$

and

$$e_2 = 1-e_1\ .$$

On the other hand,

$$p_1 = \frac{e_1 - \Phi_1}{1-\Phi_1-\Phi_2} \tag{3.2}$$

and

$$p_2 = \frac{e_2 - \Phi_2}{1-\Phi_1-\Phi_2} \qquad \text{or} \qquad p_2 = 1-p_1.$$

Suppose from a random sample of n unlabeled remotely sensed data points, n_1 points were classified into π_1 and $n_2 = n-n_1$ points were classified into π_0 by the classifier. Then

$$\hat{e}_1 = \frac{n_1}{n}\ , \tag{3.3}$$

$$\hat{p}_1 = \frac{\hat{e}_1 - \Phi_1}{1-\Phi_1-\Phi_2} \tag{3.4}$$

if Φ_1 and Φ_2 are known, and

$$\hat{\hat{p}}_1 = \frac{\hat{e}_1 - \hat{\Phi}_1}{1-\hat{\Phi}_1-\hat{\Phi}_2} \tag{3.5}$$

when Φ_1 and Φ_2 are unknown and have estimates $\hat{\Phi}_1$ and $\hat{\Phi}_2$ respectively. Clearly

$$\mathrm{Var}(\hat{p}_1) = \frac{1}{(1-\Phi_1-\Phi_2)^2}\ \mathrm{Var}(\hat{e}_1)\ . \tag{3.6}$$

For the estimate $\hat{\hat{p}}_1$ in (3.5), it easily follows that

$$\text{Bias}\ (\hat{\hat{p}}_1) = (e_1-\Phi_1)\ E[T-\theta] - E[T(\hat{\Phi}_1-\Phi_1)] \tag{3.7}$$

where

$$T = (1-\hat{\Phi}_1-\hat{\Phi}_2)^{-1}$$

and

$$\theta = (1-\Phi_1-\Phi_2)^{-1} .$$

However, evalution of expected values in (3.7) may be quite a difficult job and so an exact bias value may not be accessible. An evaluation of the $\text{MSE}(\hat{\hat{p}}_1)$ in its exact form is even more difficult. As such we instead consider having it in the following approximate form obtained in Appendix 1 using the δ-method. For a discussion on the method, see Rao (1965).

$$\text{MSE}(\hat{\hat{p}}_1) = \frac{1}{[1-\Phi_1-\Phi_2]^2}\ (\text{Var}(\hat{e}_1) + [1-\frac{e_1-\Phi_1}{1-\Phi_1-\Phi_2}]^2\ \text{Var}(\hat{\Phi}_1) + [\frac{e_1-\Phi_1}{1-\Phi_1-\Phi_2}]^2\ \text{Var}(\hat{\Phi}_2)) \tag{3.8}$$

or

$$\text{MSE}(\hat{\hat{p}}_1) = \frac{1}{[1-\Phi_1-\Phi_2]^2}\ [\text{Var}(\hat{e}_1) + (1-p_1)^2\text{Var}(\hat{\Phi}_1) + p_1^2\ \text{Var}(\hat{\Phi}_2)] \tag{3.9}$$

where p_1 is given by (3.2).

Sample Size

Considering simple random sampling with pixel as the sampling unit, we discuss the problem of sample size necessary to minimize the sampling cost or to achieve a desired amount of precision for the proportion estimate, given that the other is specified. Suppose total sample consists of $N = n + N_1 + N_2$ data points selected randomly, where n is the size of sample of unlabeled remotely sensed data used for estimating e_1, and N_1 and N_2 are sample sizes for ground truth data from π_1 and π_0 and are used to estimate Φ_1 and Φ_2, respectively. The estimates $\hat{e}_1$, $\hat{\Phi}_1$ and $\hat{\Phi}_2$ are all obtained as observed sample proportions and thus it follows from (3.6) and (3.9) that

$$\mathrm{Var}(\hat{p}_1) = \frac{e_1(1 - e_1)}{n(1-\Phi_1-\Phi_2)^2}$$

and

$$\mathrm{MSE}(\hat{\hat{p}}_1) = \frac{1}{(1-\Phi_1-\Phi_2)^2} \left[\frac{e_1(1-e_1)}{n} + (1-p_1)^2 \frac{\Phi_1(1-\Phi_1)}{N_1} + p_1^2 \frac{\Phi_2(1+\Phi_2)}{N_2}\right] . \qquad (3.10)$$

Suppose we want to obtain sample sizes necessary to minimize the sampling cost when $\mathrm{Var}(\hat{p}_1)$ and $\mathrm{MSE}(\hat{\hat{p}}_1)$ are specified, say each equal to or smaller than σ^2. In the case of Φ_1, Φ_2 known, the only cost involved is that of processing the remotely sensed sample data. Clearly, it will be minimum when the sample size n is the smallest integer greater than or equal to

$$\frac{e_1(1-e_1)}{(1 - \Phi_2-\Phi_2)^2\ \sigma^2} . \qquad (3.11)$$

For when Φ_1 and Φ_2 are unknown, there are two types of cost involved: one is the cost of processing the total sample data, say at the rate of c_1 dollars per data point and the other is the cost of obtaining ground truth, say at the rate of c_2 dollars per data point. Then the cost associated with a sample of size $N = n + N_1 + N_2$ is of the form:

$$C(N) = c_1 n + (c_1+c_2)\ (N_1+N_2) . \qquad (3.12$$

The purpose is to find N (i.e., n, N_1 and N_2) which minimize C(N) subject to $\mathrm{MSE}(\hat{\hat{p}}_1) \leq \sigma^2$. This is done in Appendix 2 where we derive explicit expressions for n, N_1 and N_2 in (A.9).

4. AN EXAMPLE

Certain sites in Hill County, Montana and Burke County, North Dakota were selected to investigate wheat identification performance for the ERTS-1 satellite data during 1973. For the sites in Hill County, there were three acquisition periods, covering both winter and spring wheat seasons, for which ERTS-1 labeled data were evaluated against the ground truth to ascertain wheat identification performance. In the case of the site in Burke County, there were only two acquisition periods covering the spring wheat season. For the classification identification performance results and other details, refer to Appendix 3.

Considering Φ_1 to be the omission percentage for the non-wheat data points and Φ_2 for the wheat data points, we give sample size results in Figure 1-7 for the various cases of omission percentages listed in Appendix 3, assuming different wheat proportions in the area and $\sigma_2 = .01$. Based on these results, the following conclusions are drawn:

1. Expected labeled wheat proportion, e_1, increases as the actual proportion of wheat, p_1, increases for the area, though not strictly. Though to a certain extent it depends upon the magnitude of the omission percentages for both non-wheat and wheat data points, it tends to centralize away from too low or too high values for the percentage.
2. Sample size for the unlabeled remotely sensed data first increases as the actual wheat proportion increases and then decreases later on; the point of decrease depends upon the size of the two omission percentages.

3. All sample sizes increase as the total omission rate $\Phi_1 + \Phi_2$ increases.

4. Sample size for the unlabeled remotely sensed data is much larger when Φ_1, Φ_2 are unknown compared to when these are known.

5. In the case of Φ_1, Φ_2 unknown, the sample size for the unlabeled remotely sensed data is proportional to c_2/c_1, the ratio of two types of cost.

6. Sample sizes for ground truth of wheat and non-wheat are inversely proportioned to c_2/c_1.

7. Sample size for the ground truth of wheat is larger than that for non-wheat when the expected labeled wheat proportion is below .5. Reverse is the case when such proportion is above .5. A similar trend holds against the actual wheat proportion, though not strictly.

8. Sample size for the ground truth increases as either of the two omission percentages increases when the other is held fixed.

For making a comparison of sample sizes irrespective of the wheat proportion which, in fact, is unknown, a suitable criterian is to determine the sample sizes against values for the coefficient of variation, $C.V. = \sigma/p$. Generally the wheat coverage in any area of interest is expected to be somewhere in between 1 percent and 20 percent. As such we here give sample sizes for the unlabeled remotely sensed data and the ground truth of wheat as well as non-wheat by specifying $\sigma = .01$ and considering certain C.V. values in a 5 to 50 percent range. Numerical results are presented in Table 1 for all different cases of Φ_1, Φ_2 values that arise from the wheat identification performance results given in Appendix 3. Moreover, for certain cases the sample sizes are sketched in Figure 8-14. The following conclusions are drawn:

1. All samples sizes increase as the total omission percentage $\Phi_1 + \Phi_2$ increases.
2. Except for the sample size for the ground truth of wheat, sample sizes decrease as the coefficient of variation increases. These are generally very high in numbers for the 5 percent co-efficient of variation but levels off when the co-efficient of variation is 50 percent.
3. Sample size for the unlabeled remotely sensed data increases considerably if Φ_1, Φ_2 are unknown compared to their known case.
4. Again, all sample sizes depend upon the ratio c_2/c_1 as regards the two types of cost.
5. Sample size for the ground truth of wheat is consistently larger than that of non-wheat. Also, it shows very small changes over the range of co-efficients of variations being considered here. In cases where there is a high overall omission percentage, and particularly for the non-wheat, it tends to increase as the co-efficient of variation increases.

TABLE 1: Sample sizes: n for the unlabeled remotely sensed data, N_1 for the ground truth of wheat, and N_2 for the ground truth of non-wheat when σ = .01

Coefficient of variation	Wheat proportion p_1	Omission rates Φ_1	Omission rates Φ_2	Expected labeled wheat proportion e_1	Φ_1, Φ_2 known: Sample size n	Φ_1 and Φ_2 unknown case, $c_2/c_1=5$: Sample size n	$c_2/c_1=5$: Ground truth sample sizes N_1	$c_2/c_1=5$: Ground truth sample sizes N_2	$c_2/c_1=20$: Sample size n	$c_2/c_1=20$: Ground truth sample sizes N_1	$c_2/c_1=20$: Ground truth sample sizes N_2
0.050	0.200	0.200	0.300	0.3000	8400	26884	7664	2195	42979	550	876
		0.100	0.250	0.2300	4192	12161	2832	1022	19100	2377	56
		0.150	0.100	0.3000	3734	10032	2706	569	16638	2264	476
		0.100	0.150	0.2500	3334	9266	2083	620	14319	1732	16
		0.050	0.200	0.2000	2645	7275	1295	594	11134	1059	486
		0.050	0.100	0.2200	2376	5667	974	336	8533	784	70
		0.000	0.050	0.1900	1706	2170	0	99	2574	0	3
0.100	0.100	0.200	0.300	0.2500	7500	24718	8390	1068	39712	7205	918
		0.100	0.250	0.1650	3261	10064	2971	477	15875	2520	405
		0.150	0.100	0.2250	3100	9433	2982	279	15054	2523	230
		0.100	0.150	0.1750	2567	7625	2212	293	12030	1866	247
		0.050	0.200	0.1250	1945	5346	1295	264	8308	1076	220
		0.050	0.100	0.1350	1617	4237	993	152	6518	817	125
		0.000	0.050	0.0950	953	1127	0	35	1278	0	21
0.150	0.067	0.200	0.300	0.2333	7156	23893	8610	705	38469	7410	607
		0.100	0.250	0.1433	2907	9182	2995	309	14646	2554	264
		0.150	0.100	0.2000	2845	8998	3061	184	14357	2611	157
		0.100	0.150	0.1500	2267	6991	2238	191	11105	1901	162
		0.050	0.200	0.1000	1600	4666	1275	168	7224	1069	141
		0.050	0.100	0.1067	1319	3658	984	97	5694	819	81
		0.000	0.050	0.0633	658	754	0	19	838	0	11
0.200	0.050	0.200	0.300	0.2250	6975	23460	8716	526	37816	7510	453
		0.100	0.250	0.1325	2721	8749	3003	229	13997	2568	196
		0.150	0.100	0.1875	2709	8729	3098	137	13972	2650	118
		0.100	0.150	0.1375	2109	6651	2247	141	10606	1916	120
		0.050	0.200	0.0875	1420	4214	1261	122	6647	1063	103
		0.050	0.100	0.0925	1162	3343	976	71	5243	818	60
		0.000	0.050	0.0475	502	565	0	12	620	0	7
0.250	0.040	0.200	0.300	0.2200	6864	23194	8778	419	37414	7569	362
		0.100	0.250	0.1260	2607	8481	3005	181	13597	2575	155
		0.150	0.100	0.1800	2624	8560	3118	110	13729	2673	94
		0.100	0.150	0.1300	2011	6438	2251	112	10293	1924	96
		0.050	0.200	0.0800	1309	3970	1250	96	6287	1058	81
		0.050	0.100	0.0840	1065	3146	969	56	4958	816	47
		0.000	0.050	0.0380	406	451	0	9	490	0	5
0.300	0.033	0.200	0.300	0.2167	6789	23014	8819	349	37142	7608	301
		0.100	0.250	0.1217	2530	8300	3006	150	13324	2580	129
		0.150	0.100	0.1750	2567	8444	3132	91	13561	2689	78
		0.100	0.150	0.1250	1945	6293	2253	93	10079	1929	80
		0.050	0.200	0.0750	1234	3803	1242	79	6041	1055	67
		0.050	0.100	0.0783	1000	3010	964	46	4761	815	39
		0.000	0.050	0.0317	340	375	0	7	405	0	4
0.350	0.029	0.200	0.300	0.2143	6735	22824	8847	299	36946	7635	258
		0.100	0.250	0.1186	2474	8168	3006	128	13127	2583	110
		0.150	0.100	0.1714	2526	8359	3141	78	13439	2700	67
		0.100	0.150	0.1214	1897	6167	2254	79	9923	1933	68
		0.050	0.200	0.0714	1180	3682	1236	67	5862	1052	57
		0.050	0.100	0.0743	952	2911	960	39	4616	814	33
		0.000	0.050	0.0271	293	321	0	6	344	0	3
0.400	0.025	0.200	0.300	0.2125	6694	22785	8868	261	36797	7656	225
		0.100	0.250	0.1163	2432	8069	3006	112	12977	2565	96
		0.150	0.100	0.1688	2494	8295	3148	68	13346	2708	59
		0.100	0.150	0.1188	1861	6107	2255	69	9805	1935	60
		0.050	0.200	0.0687	1139	3590	1231	58	5725	1050	50
		0.050	0.100	0.0712	916	2835	956	34	4506	813	29
		0.000	0.050	0.0237	257	280	0	5	300	0	3
0.450	0.022	0.200	0.300	0.2111	6662	22708	8885	232	36681	7672	200
		0.100	0.250	0.1144	2399	7991	3006	99	12860	2586	85
		0.150	0.100	0.1667	2470	8244	3153	61	13272	2714	52
		0.100	0.150	0.1167	1833	6044	2255	61	9712	1937	53
		0.050	0.200	0.0667	1107	3516	1227	52	5618	1048	44
		0.050	0.100	0.0689	868	2775	954	30	4419	812	26
		0.000	0.050	0.0211	229	245	0	4	265	0	2
0.500	0.020	0.200	0.300	0.2100	6630	22645	8898	209	36583	7684	180
		0.100	0.250	0.1130	2373	7928	3006	89	12766	2587	77
		0.150	0.100	0.1650	2450	8203	3157	55	13213	2719	47
		0.100	0.150	0.1150	1810	5993	2255	55	9637	1938	48
		0.050	0.200	0.0650	1061	3460	1224	46	5531	1046	40
		0.050	0.100	0.0670	866	2727	951	27	4348	811	23
		0.000	0.050	0.0190	207	223	0	3	237	0	2

5. A LARGE AREA CROP ACREAGE ESTIMATION

Our previous discussion, in essence, applies to crop acreage inventory for an agricultural area which is homogeneous in respect to agricultural practices and thus is not expected to be large enough. Since a major objective of the JSC-EOD project is to perform or estimate crop acreages for a large area using available remote sensing capabilities, we here suggest a sampling procedure to procure sample data for the purpose of estimating a large area crop acreage inventory and discuss the error analysis associated with it.

Once again, we assume that the frame is made of agricultural areas; the non-agricultural areas in the region of interest can be easily excluded by way of a monitoring system. As a first step in the sampling procedure, we suggest having a geographical-based stratification which effects a division of the region into reasonably homogeneous areas with respect to physical and climatological conditions. Considering additional factors of (i) the predominance of various crop-types and (ii) the latitude and longitude, a finer stratification must be achieved. This is to obtain better discrimination for the underlying crop-types and to control variability which may otherwise dominate over the distinction that exists between the resolution classes for these crop-types.

Note that as a result of stratification one may only need to consider a part of the region for frame if crops of interest do not cover the whole region. So depending upon whether the frame would require consideration of the complete region or only a part of it, one should make a list of strata making up the frame for the purpose of sampling.

Remoting sensing data gathered by an ERTS satellite is documented in terms of scenes, each covering approximately an area of 100 × 100 miles and divided into four strips where each strip has approximately 6,400 scanlines in it. As such, we suggest a three stage sampling plan to be independently carried out in each stratum: select randomly ERTS scenes at the first stage, strips within scenes at the second stage and scanlines within strips at the third stage. Of course, one may consider one more stage in selecting pixels within scanlines. However, sampling at this stage is excluded from the plan because it is inconvenient and uneconomical.

Notations

Let R be the region (in the sense of frame) of interest for estimating crop acreages. Suppose it is stratified into strata R_t, t=1,2,...,L, with weights w_t, the proportion of pixels in tth stratum, t=1,2,...,L so that

$$R = \bigcup_{t=1}^{L} R_t \qquad \text{with} \quad \sum_{t=1}^{L} w_t = 1 \ .$$

In stratum R_t, let I_t be the number of scenes whereas J, H and n denote the number of strips per scene, number of scanlines per strip and number of pixels per scanlines, respectively. From the previous paragraph it is obvious that there is no need to distinguish between strata in the categories of strips per scene, scanlines per strip and pixels per scanline. Next, let $e_{tijh}(\pi_k)$ be the expected proportion of pixels to be classified in π_k from the hth scanline in jth strip of ith scene for stratum R_t, t=1,2,...,L. Then for R_t,

$$e_{tij}(\pi_k) = \sum_{h=1}^{H} e_{tijh}(\pi_k)$$

the expected proportion of pixels to be classified in π_k from the jth strip in ith scene,

$$e_{ti}(\pi_k) = \sum_{j=1}^{J} \sum_{h=1}^{H} e_{tijh}(\pi_k),$$

the expected proportions of pixels to be classified in π_k from the ith scene,

$$e_t(\pi_k) = \sum_{i=1}^{I_t} \sum_{j=1}^{J} \sum_{h=1}^{H} e_{tijh}(\pi_k) ,$$

the expected proportion of pixels to be classified in π_k. Accordingly,

$$e(\pi_k) = \sum_{t=1}^{L} w_t e_t(\pi_k) , \tag{5.1}$$

is the expected proportion of pixels to be classified in π_k, $k=1,2,\ldots,m$, for the region R.

Estimates

Suppose m_t, r and s denote the corresponding number of scenes, number of strips per scenes and number of scanlines per strip that one selected for R_t, $t=1,2,\ldots,L$, using the stratified three stage random sampling described earlier. Let $n_{tijh}(\pi_k)$ be the number of pixels classified into π_k from the hth selected scanline in jth selected strip of the ith selected scene in R_t. Then considering the observed proportions of classified data points into different crops for estimates, one has

$$\hat{e}_{tijh}(\pi_k) = \frac{n_{tijh}(\pi_k)}{n} ,$$

$$\hat{e}_{tij}(\pi_k) = \frac{1}{ns} \sum_{h=1}^{s} n_{tijh}(\pi_k),$$

$$\hat{e}_{ti}(\pi_k) = \frac{1}{nsr} \sum_{j=1}^{r} \sum_{h=1}^{s} n_{tijh}(\pi_k) ,$$

$$\hat{e}_t(\pi_k) = \frac{1}{nsrm_t} \sum_{i=1}^{m_t} \sum_{j=1}^{r} \sum_{h=1}^{s} n_{tijh}(\pi_k) ,$$

and

$$\hat{e}(\pi_k) = \sum_{t=1}^{L} w_t \hat{e}_t(\pi_k) , \qquad k = 1,2,\ldots,m . \tag{5.2}$$

Next, expressions for $Var(e_t(\pi_k))$ and $Cov(e_t(\pi_k), e_t(\pi_k))(k \neq k')$ can be obtained without much difficulty. For example, refer to Section 10.8 in Cochran (1963) for the general discussion on three stage sampling plan. Hence, the covariance matrix of $\hat{e}$ is given by

$$Var(\hat{e}(\pi_k)) = \sum_{t=1}^{L} w_t^2 \; Var(\hat{e}_t(\pi_k))$$

and

$$Cov(\hat{e}(\pi_k), \hat{e}(\pi_{k'})) = \sum_{t=1}^{L} w_t^2 \, Cov(\hat{e}_t(\pi_k), \hat{e}_t(\pi_{k'})), \; k \neq k', \quad k=1,2,\ldots,m . \tag{5.3}$$

Similarly, an estimate of the covariance matrix is obtained by replacing the unknown quantities by their estimates in (5.3). In this context, see Chhikara and Odell (1974) who have discussed such results in greater details.

Now to obtain the actual crop proportions, there is a need to consider whether or not the classification error matrix is the same for each stratum. When the area is wide and large and the stratification is performed considering factors mentioned in the beginning of this section, it is quite likely that these classification error matrices will not be the same for different strata.

In that case, find an estimate of p_k, k=1,2,...,m, using (2.5) if the classification error matrix is known and (2.6) if it is unknown for each stratum. Denoting p_k by $p_t(\pi_k)$ for stratum R_t, it then follows that

$$\hat{p}_k = \sum_{t=1}^{L} w_t \hat{p}_t(\pi_k) \quad , \qquad k = 1,2,\ldots,m \tag{5.4}$$

when the classification matrices, say P_t, t = 1,2,...,L, are known, and

$$\hat{\hat{p}}_k = \sum_{t=1}^{L} w_t \hat{\hat{p}}_t(\pi_k) \quad , \qquad k = 1,2,\ldots,m \tag{5.5}$$

when these are unknown and are separately estimated using ground truth data from each stratum. Next, Var($\hat{p}_k$) and MSE($\hat{\hat{p}}_k$) are respectively obtained from (A.2) and (A.7) after making an appropriate substitution from (5.2).

On the other hand, either there is the same classification error matrix for all strata or can be made so by proper adjustment of signatures in the classification algorithm for each stratum. For then an estimate of crop proportions p_k, k=1,2,...,m is directly given by (2.5) if the common classification error matrix is known and by (2.6) if it is unknown, using $\hat{e}(\pi_k)$, k=1,2,...,m of (5.1) for e. Hence, both estimates and their error analyses are obtained by following the general procedure given in Section 2.

In fact, our approach in Section 2 is quite general and can be applied to perform any large area acreage inventory by considering an appropriate sampling scheme for both the unlabeled remotely sensed data and the ground truthed data.

Once again if interest lies in estimating only the wheat acreage, the two-crop approach of Section 3 can be applied. Then an estimate of wheat proportion is obtained from (3.4) or (3.5) as the case may be, either first obtaining it stratumwise and then combining as we did above in (5.4) and (5.5) or directly, depending upon whether or not the classification error matrix is the same for different strata. Subsequently, the precision of this estimate and the sample size necessary to achieve a desired precision with minimum cost can be easily obtained by applying our technique of Section 3.

Sample Size

Taking the cost factor into consideration, suppose we want to determine the sample size that either minimizes the sampling cost for a specified precision or maximizes the precision of the estimate for a fixed cost. Though a large initial cost is involved in acquiring remotely sensed data, presently we are mainly concerned with the cost of the processing and labeling of the sampled data. In general, any such cost can be considered as

$$C_t = c_1 m_t + c_2 m_t r + c_3 m_t rs$$

for the sample in stratum R_t, and

$$C = (c_1 + c_2 r + c_3 rs) \sum_{t=1}^{L} m_t$$

for the area of interest.

In case of unknown classification error matrix or matrices, there is an additional cost of sampling the ground truth, say C'. As such the total cost involved is $C = C + C'$. Now if the cost is fixed, say $C'' \leq C_0$, a determination of sample sizes for both the unlabeled remotely sensed data in all three categories and the ground truth for various crops can be achieved by solving equations obtained by equating the partial derivatives of

$$\mathrm{MSE}(\hat{\hat{p}}_k) + \lambda(C''-C_0) , \qquad k = 1,2,\ldots,m$$

where λ is a Lagrange multiplier, with respect to m_t, r, s and the ground truth sample sizes to zero. Similarly, when the MSE $(\hat{\hat{p}}_k)$ is fixed, say σ_k^2, k=1,2,...,m, again this can be achieved by considering the function

$$C'' + \lambda_k[\mathrm{MSE}(\hat{\hat{p}}_k) - \sigma_k^2], \quad k = 1,2,\ldots,m$$

for minimization. This, of course, would lead to k different values for various sample sizes unless we consider the minimization from the point of a specific crop-type proportion estimate. On the other hand, a unique determination can be obtained by considering the largest value obtained in each case.

It may be pointed out that under this procedure, it will be difficult to give any closed form expression for any sample size and its carrying out would involve some optimization technique.

If the classification error matrix (or matrices) is known, the sample sizes m_t(t=1,2,...,L), r and s can be easily determined by minimizing $\mathrm{Var}(\hat{e}(\pi_k)) + \lambda(C-C_0)$ or $C + \lambda_k[\mathrm{Var}(\hat{e}(\pi_k)) - \sigma_k^2]$ as the case may be. Moreover, the sample size problem in the case of unknown classification error matrix or matrices can be treated either by assuming the classification errors known or by investigating the two types of sampling separately.

6. FURTHER REMARKS

In actual practice it may not be possible to have every data point identified with one of the crops in the area of interest, particularly if the area is large. This may be caused by not knowing all crop-types that exist in the area or some data points representing pixels falling on the field boundaries. As such the model developed in this report may be viewed somewhat restricted. Its use for performing a large area crop inventory may be considered subsequent to obtaining information about the agricultural practices in the area.

It is extremely difficult to model the problem of a large area crop inventory in its full generality unless certain constraints are imposed. The condition of identifiability is one such constraint that one must have in order to deal with the problem analytically.

APPENDIX 1

A.1. Variances of Components of $\hat{p}$

For $\hat{p}$ given in (2.5), the covariance matrix,

$$E[(\hat{p} - p)(\hat{p} - p)^T] = P^{-1} E[(\hat{e} - e)(\hat{e} - e)^T](P^{-1})^T$$

or

$$\Sigma_{\hat{p}} = (P^{-1}) \; V \; (P^{-1})^T \qquad (A.1)$$

where V denotes the covariance matrix of $\hat{e}$. Denoting the $(i, j)^{th}$ element of P^{-1} by p^{ij}, it follows that the variance of $\hat{p}_i$, the ith element of $\hat{p}$, is given by

$$\text{Var}\ (\hat{p}_i) = \sum_{j=1}^{m} (p^{ij})^2 \ \text{Var}\ (\hat{e}_j) + \sum_{j=1}^{m} \sum_{\substack{k=1 \\ j \neq k}}^{m} p^{ij}\, p^{ik} \ \text{Cov}\ (\hat{e}_j, \hat{e}_k) \qquad (A.2)$$

where $V(\hat{e}_j)$ and $\text{Cov}\ (\hat{e}_j, \hat{e}_k)$ would depend upon the sampling scheme used for obtaining samples of unlabeled remotely sensed data points.

In the case of random sampling with sampling unit as pixel (i.e. one data point),

$$\text{Var}\ (\hat{e}_j) = \frac{e_j(1 - e_j)}{n} \qquad (A.3)$$

and

$$\text{Cov}\ (\hat{e}_j, \hat{e}_k) = -\frac{e_j e_k}{n}, \ j \neq k, \ j, k = 1, 2, .., m,$$

ignoring the finite population correction due to large population size. Next an unbiased estimate of these quantities is given by

$$\widehat{\text{Var}}\ (\hat{e}_j) = \frac{\hat{e}_j(1 - \hat{e}_j)}{n - 1}$$

$$\widehat{\text{Cov}}\ (\hat{e}_j, \hat{e}_k) = -\frac{\hat{e}_j\ \hat{e}_k}{n - 1},\ j \neq k,\ j,\ k = 1,\ 2,\ ..,\ m.$$

On the other hand if the sampling unit is a 5 x 6 mile segment consisting of r pixels then considering a random sample of m segments (here for the sample size one may consider $n = mr$ data points) from the total of M segments in the area of interest and again ignoring the finite population correction, one gets (Cochran, 1963)

$$\text{Var}\ (\hat{e}_j) = \frac{1}{m(M - 1)} \sum_{i=1}^{M} (e_{ji} - e_j)^2$$

and

$$\text{Cov}\ (\hat{e}_j, \hat{e}_k) = \frac{1}{m\ (M-1)} \sum_{i=1}^{M} (e_{ji} - e_j)(e_{ki} - e_k),\ j \neq k \qquad \text{(A.4)}$$

$$j,\ k = 1,\ 2,\ ..,\ m$$

where e_{ji} denotes the proportion of classified data points in π_j for the ith segment. Once again, for their unbiased estimates

$$\widehat{\text{Var}}\ (\hat{e}_j) = \frac{1}{m(m - 1)} \sum_{i=1}^{m} (\hat{e}_{ji} - \hat{e}_j)^2$$

and

$$\widehat{\text{Cov}}\,(\hat{e}_j, \hat{e}_k) = \frac{1}{m(m-1)} \sum_{i=1}^{m} (\hat{e}_{ji} - \hat{e}_j)(\hat{e}_{ki} - \hat{e}_k), \; i \neq k,$$

$$j, k = 1, 2, \ldots, m.$$

Similarly, components of V and their estimates can be obtained for other types of sampling plans. Making appropriate substitution in (A.1) or (A.2), variances for the components of $\hat{p}$ and their estimates are then obtained.

A.2. Mean Square Errors of Components of $\hat{\hat{p}}$.

First we calculate the bias of $\hat{\hat{p}}$ given by

$$\begin{aligned} \text{Bias}\,(\hat{\hat{p}}) &= E[\hat{\hat{p}}-p] \\ &= E\,[\hat{P}^{-1}\hat{e} - P^{-1}e] \\ &= E\,[\hat{P}^{-1}(\hat{e}-e) + (\hat{P}^{-1} - P^{-1})e] \\ &= E\,[\hat{P}^{-1} - P^{-1}]e \qquad \text{(A.5)} \end{aligned}$$

because the first term is zero due $E(\hat{e}-e) = 0$ for a given $\hat{P}^{-1}$. Clearly, the bias depends upon how much bias there is in $\hat{P}^{-1}$, and

$$\text{Bias}\,(\hat{\hat{p}}) = (\text{Bias}\,(\hat{P}^{-1}))e.$$

In order to find the mean square error of any component of $\hat{\hat{p}}$, let us first consider the evaluation of matrix,

$$E\,[(\hat{\hat{p}}-p)(\hat{\hat{p}}-p)^T] = E\,[(\hat{P}^{-1}\hat{e} - P^{-1}e)(\hat{P}^{-1}\hat{e} - P^{-1}e)^T]$$

so

$$= E\,[(\hat{P}^{-1})(\hat{e}-e)(\hat{e}-e)^T(\hat{P}^{-1})^T + (\hat{P}^{-1} - P^{-1})\, ee^T\, (\hat{P}^{-1} - P^{-1})^T]$$

$$= E\,[(\hat{P}^{-1})\, V\, (\hat{P}^{-1})^T] + E\,[(\hat{P}^{-1} - P^{-1})\, ee^T\, (\hat{P}^{-1} - P^{-1})^T] \tag{A.6}$$

where E stands for expectation with respect to $\hat{P}$. Again, denoting the (i, j)th element of P^{-1} by P^{ij} and that of $\hat{P}^{-1}$ by $\hat{P}^{ij}$, it follows from (A.6) that the mean square error of $\hat{p}_i$, the i^{th} component of $\hat{p}$, is given by

$$\mathrm{MSE}\,(\hat{\hat{p}}_i) = E\Bigg[\sum_{j=1}^{m} (\hat{P}^{ij})^2\, \mathrm{Var}\,(\hat{e}_j) + \sum_{\substack{j=1\\ j\neq k}}^{m}\sum_{k=1}^{m} \hat{P}^{ij}\, \hat{P}^{ik}\, \mathrm{Cov}\,(\hat{e}_j, \hat{e}_k)$$

$$+ \sum_{j=1}^{m} e_j^2\, E\,[\hat{P}^{ij} - P^{ij}]^2 + \sum_{\substack{j=1\\ j\neq k}}^{m}\sum_{k=1}^{m} e_j e_k E[(\hat{P}^{ij}-P^{ij})(\hat{P}^{ik}-P^{ik})]\Bigg], \tag{A.7}$$

$i = 1, 2, \ldots, m.$

Once again, V, i.e. $\mathrm{Var}\,(\hat{e}_j)$ and $\mathrm{Cov}\,(\hat{e}_j, \hat{e}_k)$, j and $k = 1, 2, \ldots, m$, may be obtained as in (A.3) and (A.4). If some other sampling plan is used for selecting remotely sensed data to obtain the estimates $\hat{e}_j$'s, expression for V can accordingly be obtained. To evaluate expectation in (A.7), one needs to find the distribution of $\hat{P}$. This will, of course, depend upon how $\hat{P}$ is obtained. In general, it will be difficult to obtain any exact distribution of $\hat{P}$. However, if the sampling of ground truth involves separate independent samples from each crop and $\hat{P}$ is obtained as the matrix of observed proportions among randomly selected pixels classified into different crops using a classifier, each column vector of $\hat{P}$ has a multinomial distribution and is stochaotically independent of the others in $\hat{P}$. Since expectation in (A.7) is for elements of $\hat{P}^{-1}$, it may not be easy to derive the MSE $(\hat{p}_i)$ in a closed form, especially if the number of crops is large.

APPENDIX 2

Two-Crop Case

First we derive the MSE($\hat{p}_1$) as in (3.8).

Proof of (3.8)

Considering the estimates $\hat{\Phi}_1$, $\hat{\Phi}_2$ and $\hat{e}$, being obtained from independent sets of samples, it follows by an application of the δ-method that

$$MSE(\hat{p}_1) \doteq \left(\frac{\partial p_1}{\partial e_1}\right)^2 Var(\hat{e}_1) + \left(\frac{\partial p_1}{\partial \Phi_1}\right)^2 Var(\hat{\Phi}_1) + \left(\frac{\partial p_1}{\partial \Phi_2}\right)^2 Var(\hat{\Phi}_2)$$

$$\doteq \frac{1}{(1-\Phi_1-\Phi_2)^2} Var(\hat{e}_1) + \left[\frac{e_1-1+\Phi_2}{(1-\Phi_1-\Phi_2)^2}\right]^2 Var(\hat{\Phi}_1)$$

$$+ \left[\frac{e_1-\Phi_1}{(1-\Phi_1-\Phi_2)^2}\right]^2 Var(\hat{\Phi}_2) \quad .$$

Hence

$$MSE(\hat{p}_1) \doteq \frac{1}{(1-\Phi_1-\Phi_2)^2} \left(Var(\hat{e}_1) + \left[1 - \frac{e_1-\Phi_1}{1-\Phi_1-\Phi_2}\right]^2 Var(\hat{\Phi}_1) \right.$$

$$\left. + \left[\frac{e_1-\Phi_1}{1-\Phi_1-\Phi_2}\right]^2 Var(\hat{\Phi}_2) \right) .$$

Here dot with equality sign means equality with approximation. This establishes (3.8).

For a determination of sample size necessary to minimize the cost subject to MSE ($\hat{p}_1$) $\leq \sigma^2$ as discussed in section 3, it is achieved by minimizing the function

$$F = C(N) + \lambda \left(MSE(\hat{\hat{p}}_1) - \sigma^2\right)$$

with respect to n, N_1 and N_2, where C (N) is given in (3.12) and $MSE(\hat{\hat{p}}_1)$ is given in (3.10). By rewriting, we have

$$F = c_1 n + (c_1 + c_2)(N_1 + N_2) + \lambda (1 - \Phi_1 - \Phi_2)^{-2}$$
$$\cdot \left[\frac{e_1(1-e_1)}{n} + (1-p_1)^2 \frac{\Phi_1(1-\Phi_1)}{N_1} + p_1^2 \frac{\Phi_2(1-\Phi_2)}{N_2} - \sigma^2(1-\Phi_1-\Phi_2)^2 \right].$$

Taking partial derivatives of F with respect to n, N_1 and N_2 and equating each to zero, one obtains the following set of equations.

$$c_1 - \lambda(1-\Phi_1-\Phi_2)^{-2} \frac{e_1(1-e)}{n^2} = 0$$

$$(c_1 + c_2) - \lambda(1-\Phi_1-\Phi_2)^{-2} (1-p_1)^2 \frac{\Phi_1(1-\Phi_1)}{N_1^2} = 0$$

$$(c_1 + c_2) - \lambda(1-\Phi_1-\Phi_2)^{-2} p_1^2 \frac{\Phi_2(1-\Phi_2)}{N_2^2} = 0 \quad .$$

Considering only the admissible solution of these equations, one has

$$n = \sqrt{\frac{\lambda e_1(1-e_1)}{c_1(1-\Phi_1-\Phi_2)^2}}$$

$$N_1 = (1-p_1)\sqrt{\frac{\lambda \Phi_1(1-\Phi_1)}{(c_1+c_2)(1-\Phi_1-\Phi_2)^2}} \qquad \text{(A.8)}$$

$$N_2 = p_1\sqrt{\frac{\lambda \Phi_2(1-\Phi_2)}{(c_1+c_2)(1-\Phi_1-\Phi_2)^2}} \quad .$$

Considering that MSE $(\hat{\hat{p}}_1) \doteq \sigma^2$ and making substitution in (3.10) for n, N_1 and N_2 obtained in (A.8), one gets

$$\sqrt{\lambda} = \frac{1}{\sigma^2(1-\phi_1-\phi_2)}\left[\sqrt{c_1e_1(1-e_1)} + (1-p_1)\sqrt{(c_1+c_2)\,\phi_1(1-\phi_1)} + p_1\sqrt{(c_1+c_2)\phi_2(1-\phi_2)}\right].$$

After substituting for $\sqrt{\lambda}$ in (A.8), the sample sizes n, N_1 and N_2 are obtained as following:

$$n = \sqrt{\frac{e_1(1-e_1)/c_1}{\sigma^2(1-\phi_1-\phi_2)^2}}\left[\sqrt{c_1e_1\,(1-e_1)} + (1-p_1)\sqrt{(c_1+c_2)\phi_1(1-\phi_1)} + p_1\sqrt{(c_1+c_2)\phi_2(1-\phi_2)}\right]$$

$$N_1 = \frac{(1-p_1)}{\sigma^2(1-\phi_1-\phi_2)^2}\sqrt{\frac{\phi_1(1-\phi_1)}{c_1+c_2}}\left[\sqrt{c_1e_1(1-e_1)} + (1-p_1)\sqrt{(c_1+c_2)\,\phi_1(1-\phi_1)} + p_1\sqrt{(c_1+c_2)\phi_2(1-\phi_2)}\right]$$

$$N_2 = \frac{p_1}{\sigma^2(1-\phi_1-\phi_2)^2}\sqrt{\frac{\phi_2(1-\phi_2)}{c_1+c_2}}\left[\sqrt{c_1e_1(1-e_1)} + (1-p_1)\sqrt{(c_1+c_2)\phi_1(1-\phi_1)} + p_1\sqrt{(c_1+c_2)\phi_2(1-\phi_2)}\right]. \quad (A.9)$$

It can be easily seen that n is a monotone increasing and N_1, N_2 are monotone decreasing functions in c_2/c_1, the ratio of two types of cost. For when e_1, ϕ_1, ϕ_2, are unknown, estimates of n, N_1 and N_2 can be obtained from (A.9) by replacing these unknown quantities by their estimates.

APPENDIX 3

ERTS-1 DATA INVESTIGATION

FOR

WHEAT IDENTIFICATION

1. Hill County, Montana

- Complete ground for evaluation in 2 × 6 mile area in Hill County North

- Ground identifications of wheat, barley, oats in Hill County South

- ERTS-1 data evaluated at three acquisition periods covering spring and winter wheat seasons

Date	Winter Wheat Stage	Spring Wheat Stage
May 23	Greening	Pre-emergence
June 27	Heading	100% cover
July 16	Mature	Headed

- Classification performance results:

W - Spring/Winter Wheat
NW - Oats/Barley/Pasture

Commission/Omission Percentages

$$\begin{array}{c} \\ W \\ NW \end{array}\begin{array}{c} \begin{array}{cc} W & NW \end{array} \\ \begin{bmatrix} 70 & 30 \\ 20 & 80 \end{bmatrix} \end{array}$$

May 23(t_1)

$$\begin{array}{c} \\ W \\ NW \end{array}\begin{array}{c} \begin{array}{cc} W & NW \end{array} \\ \begin{bmatrix} 90 & 10 \\ 15 & 85 \end{bmatrix} \end{array}$$

June 27(t_2)

$$\begin{array}{c} \\ W \\ NW \end{array}\begin{array}{c} \begin{array}{cc} W & NW \end{array} \\ \begin{bmatrix} 80 & 20 \\ 5 & 95 \end{bmatrix} \end{array}$$

July 16(t_3)

$$\begin{array}{c} \\ W \\ NW \end{array}\begin{array}{c} \begin{array}{cc} W & NW \end{array} \\ \begin{bmatrix} 90 & 10 \\ 5 & 95 \end{bmatrix} \end{array}$$

May, June
(t_1,t_2)

$$\begin{array}{c} \\ W \\ NW \end{array}\begin{array}{c} \begin{array}{cc} W & NW \end{array} \\ \begin{bmatrix} 90 & 10 \\ 5 & 95 \end{bmatrix} \end{array}$$

May, July
(t_1,t_3)

$$\begin{array}{c} \\ W \\ NW \end{array}\begin{array}{c} \begin{array}{cc} W & NW \end{array} \\ \begin{bmatrix} 95 & 5 \\ 0 & 100 \end{bmatrix} \end{array}$$

May, June, July
(t_1,t_2,t_3)

2. Burke County, North Dakota

- Complete ground truth for evaluation in 2 × 10 mile area
- ERTS-1 data evaluated at two acquisition periods

Date	Spring Wheat Stage
June 5	3"-4" growth
June 23	Jointing

- Classification performance results:

W - Spring Wheat
NW - Barley/Oats/Pasture/Summer Fallow

Commission/Omission Percentages

$$\begin{array}{c|cc} & W & NW \\ \hline W & 75 & 25 \\ NW & 10 & 90 \end{array}$$

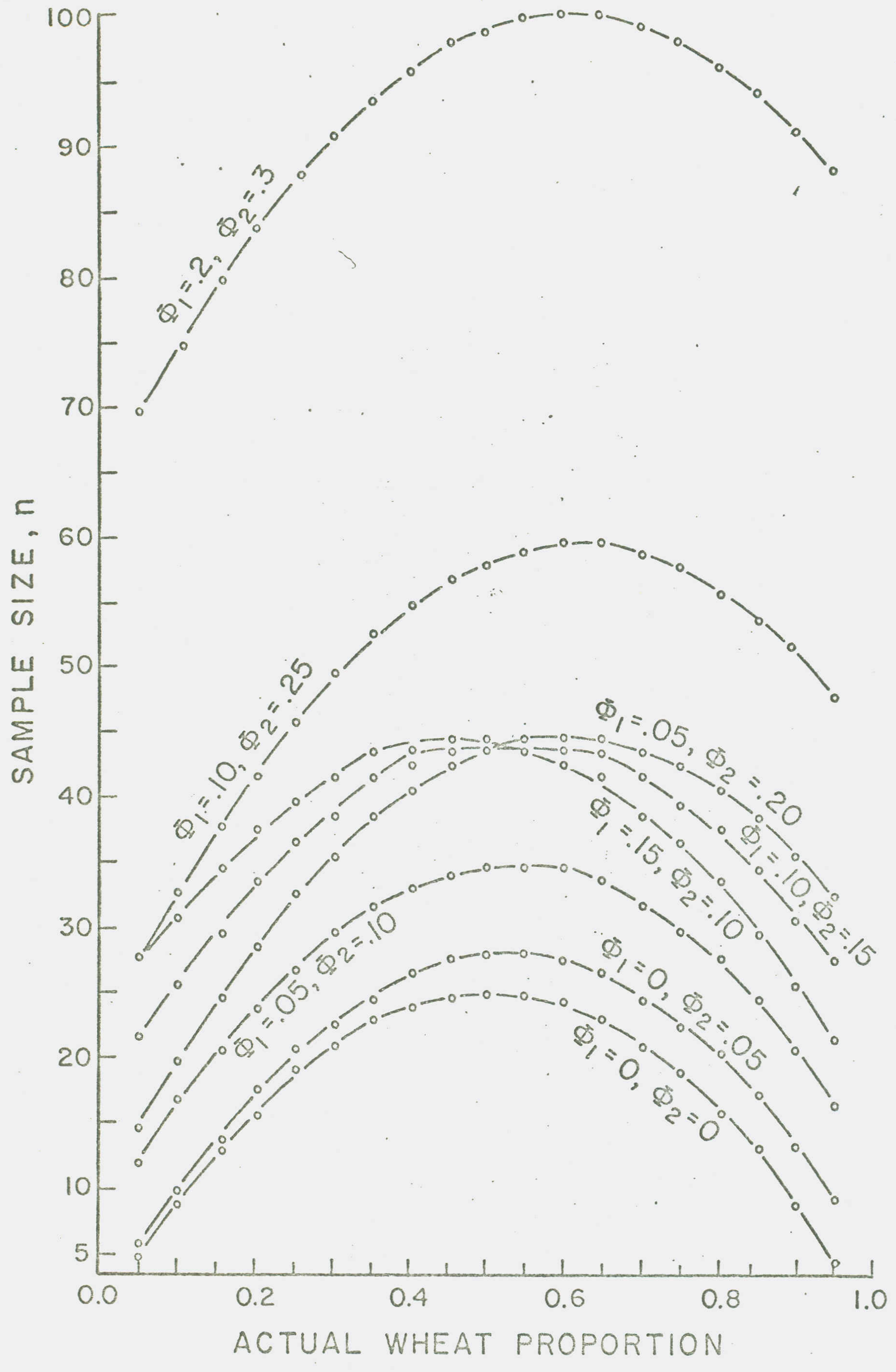
June 5(t_1)

$$\begin{array}{c|cc} & W & NW \\ \hline W & 85 & 15 \\ NW & 10 & 90 \end{array}$$

June 23(t_2)

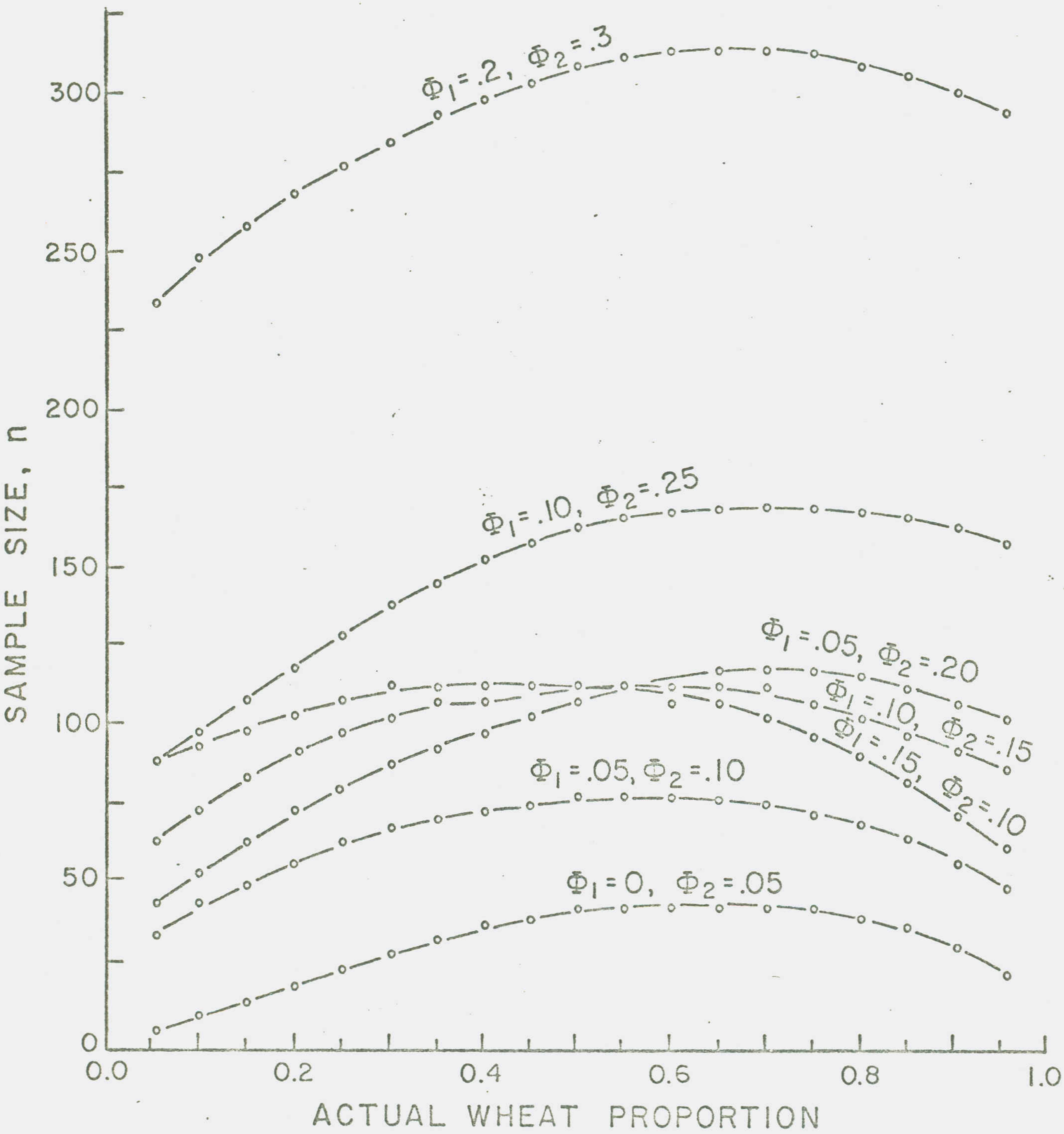
$$\begin{array}{c|cc} & W & NW \\ \hline W & 90 & 10 \\ NW & 5 & 95 \end{array}$$

June 5, June 23
(t_1,t_2)



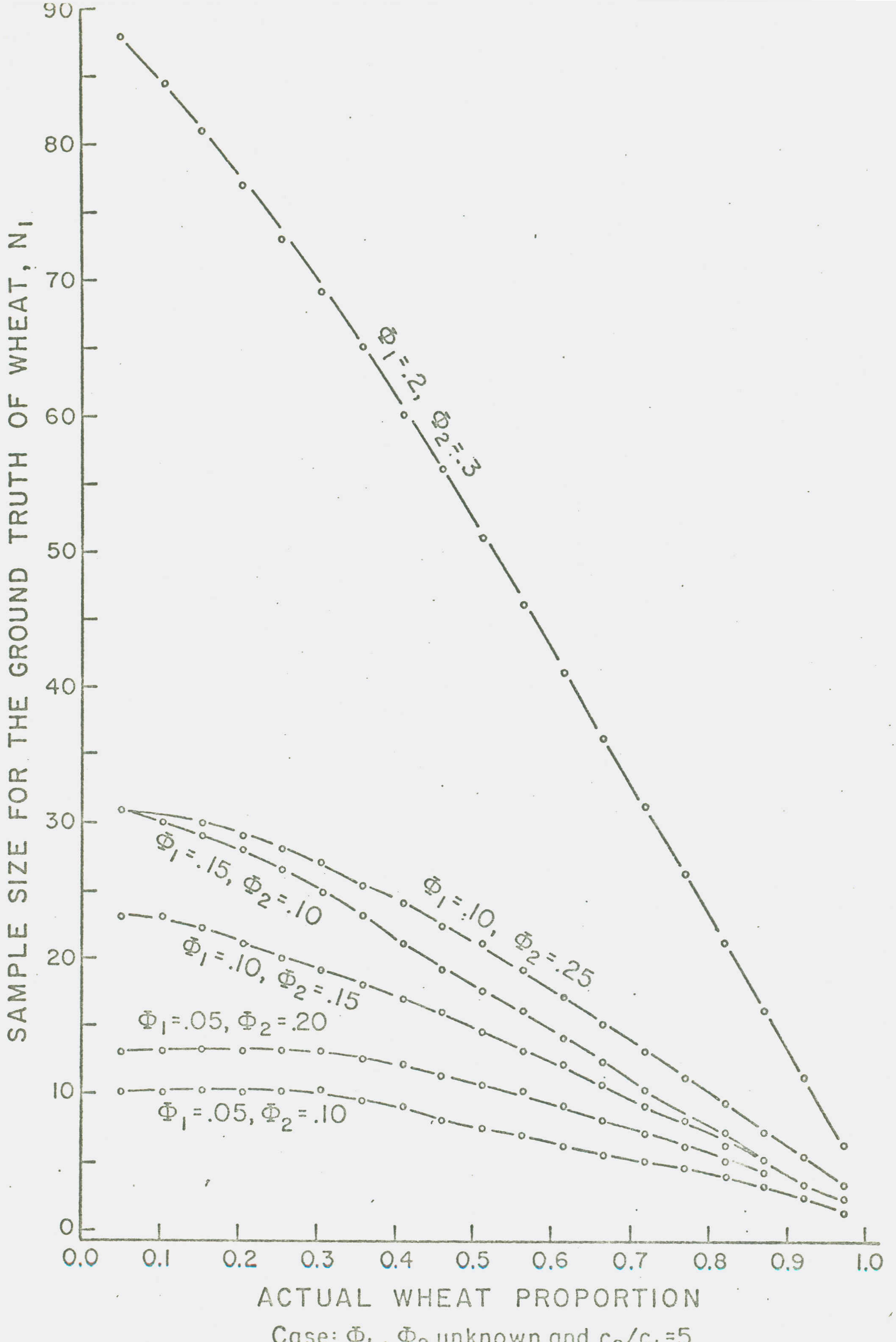
Case: Φ_1, Φ_2 known

FIGURE I



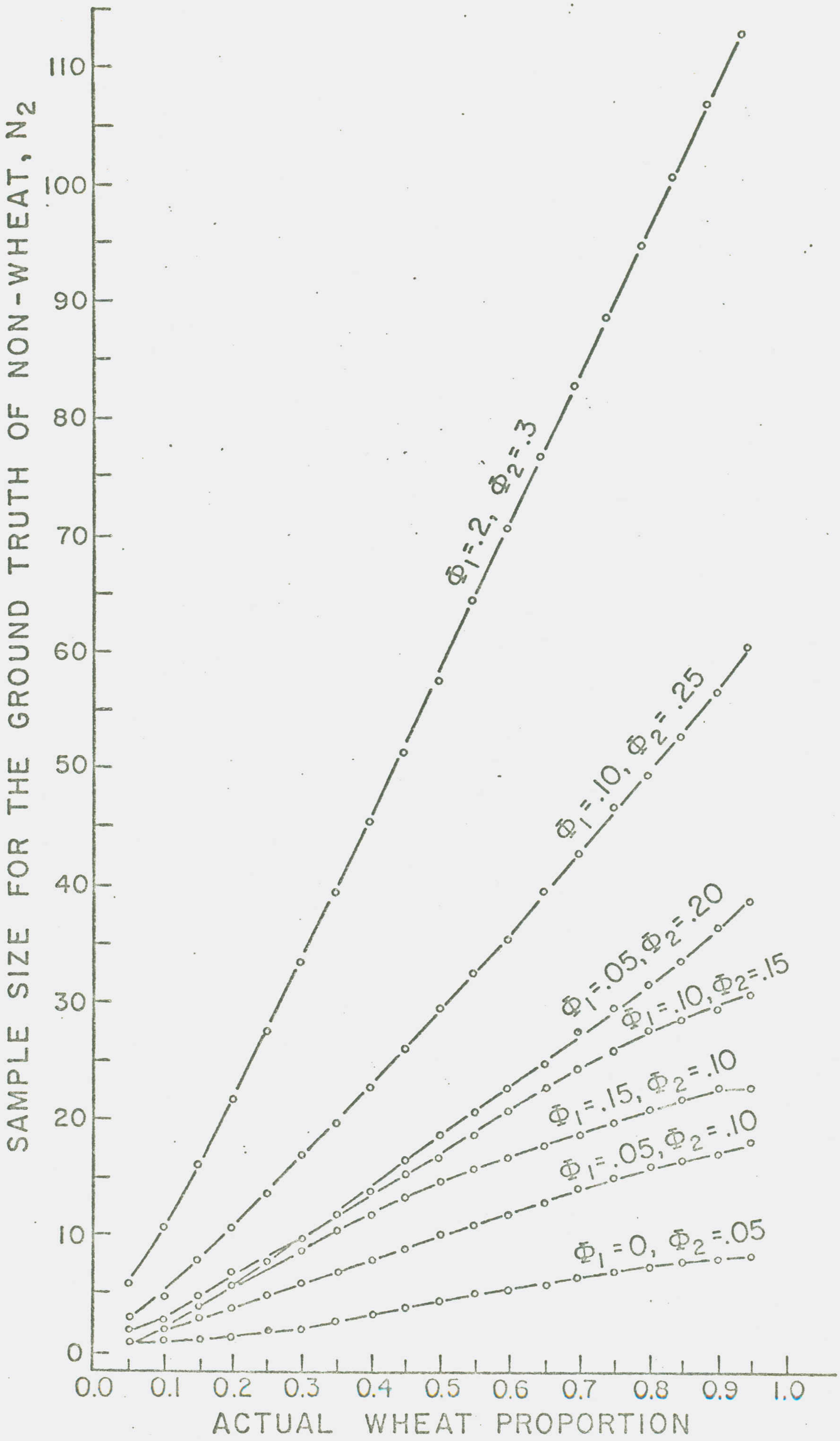
Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 5$

FIGURE 2



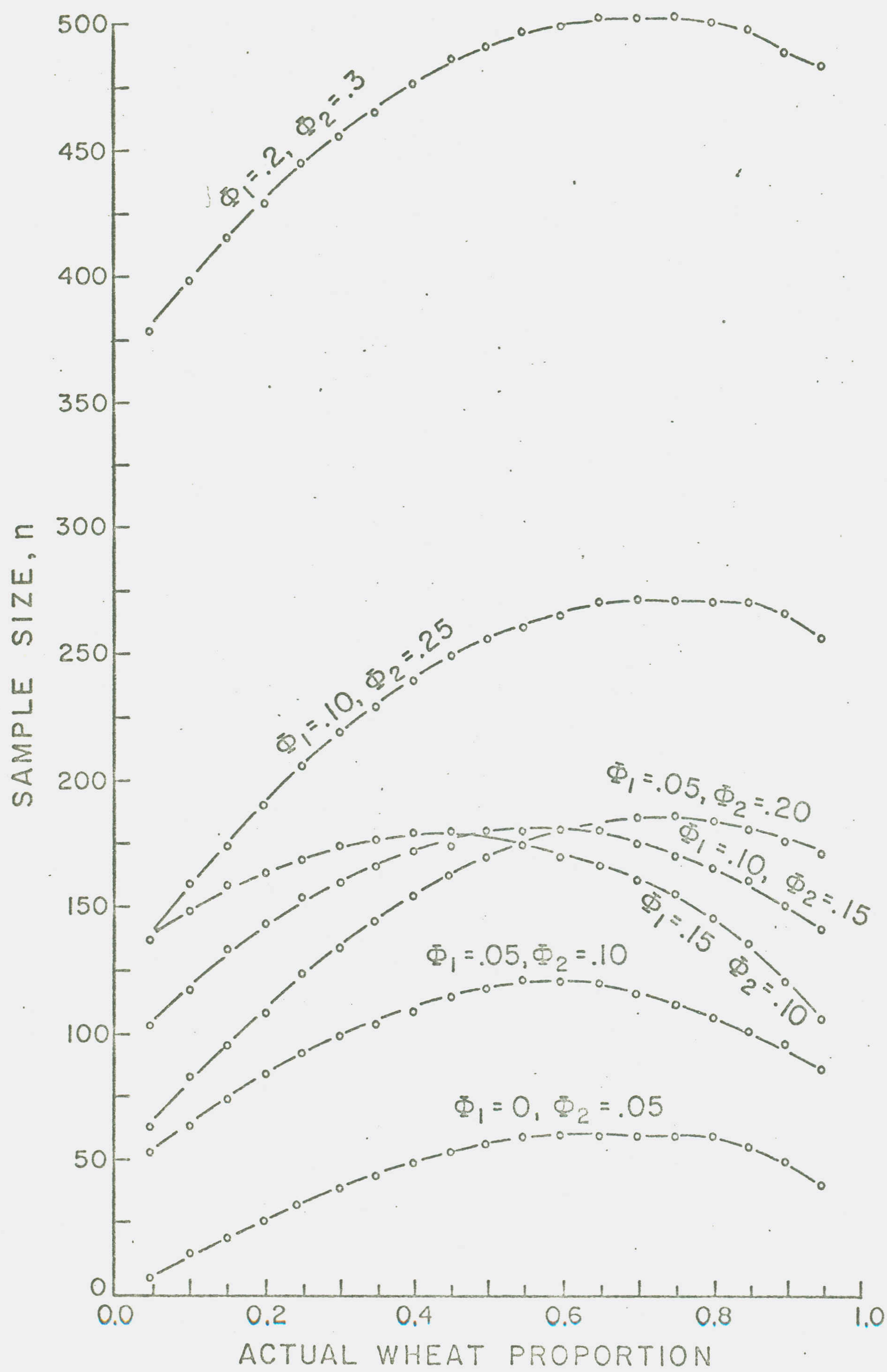
Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 5$

FIGURE 3



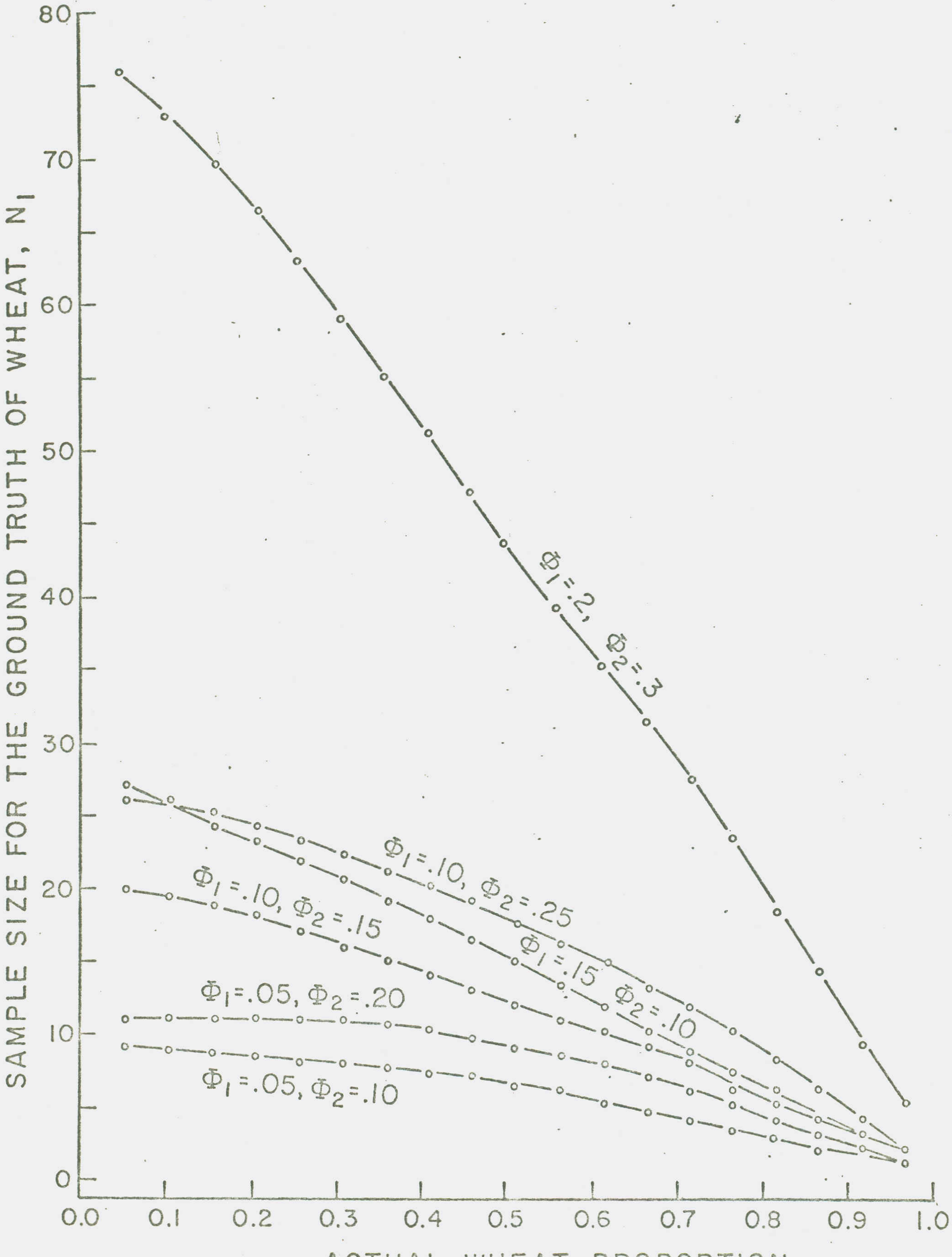
Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 5$

FIGURE 4



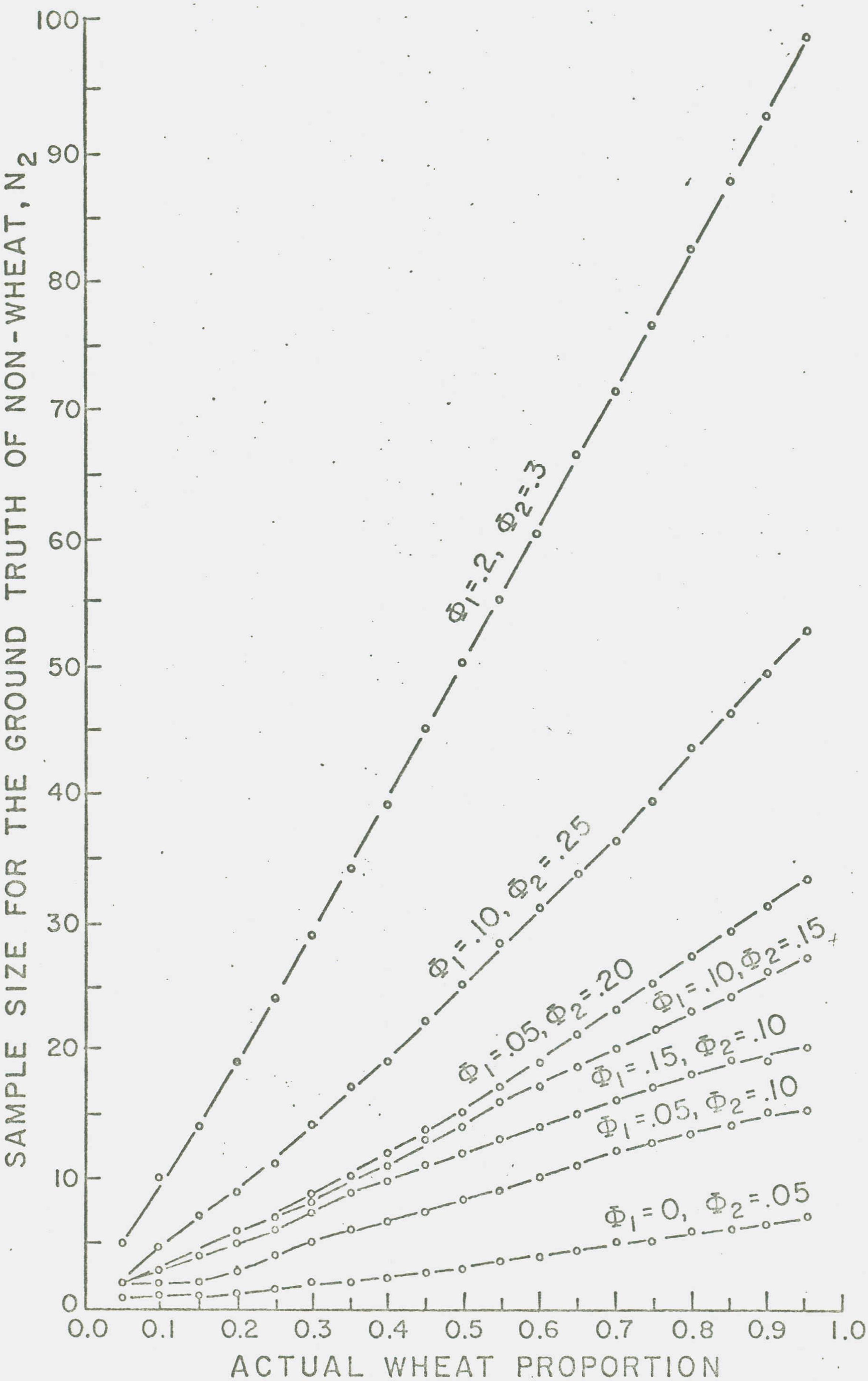
Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 20$

FIGURE 5



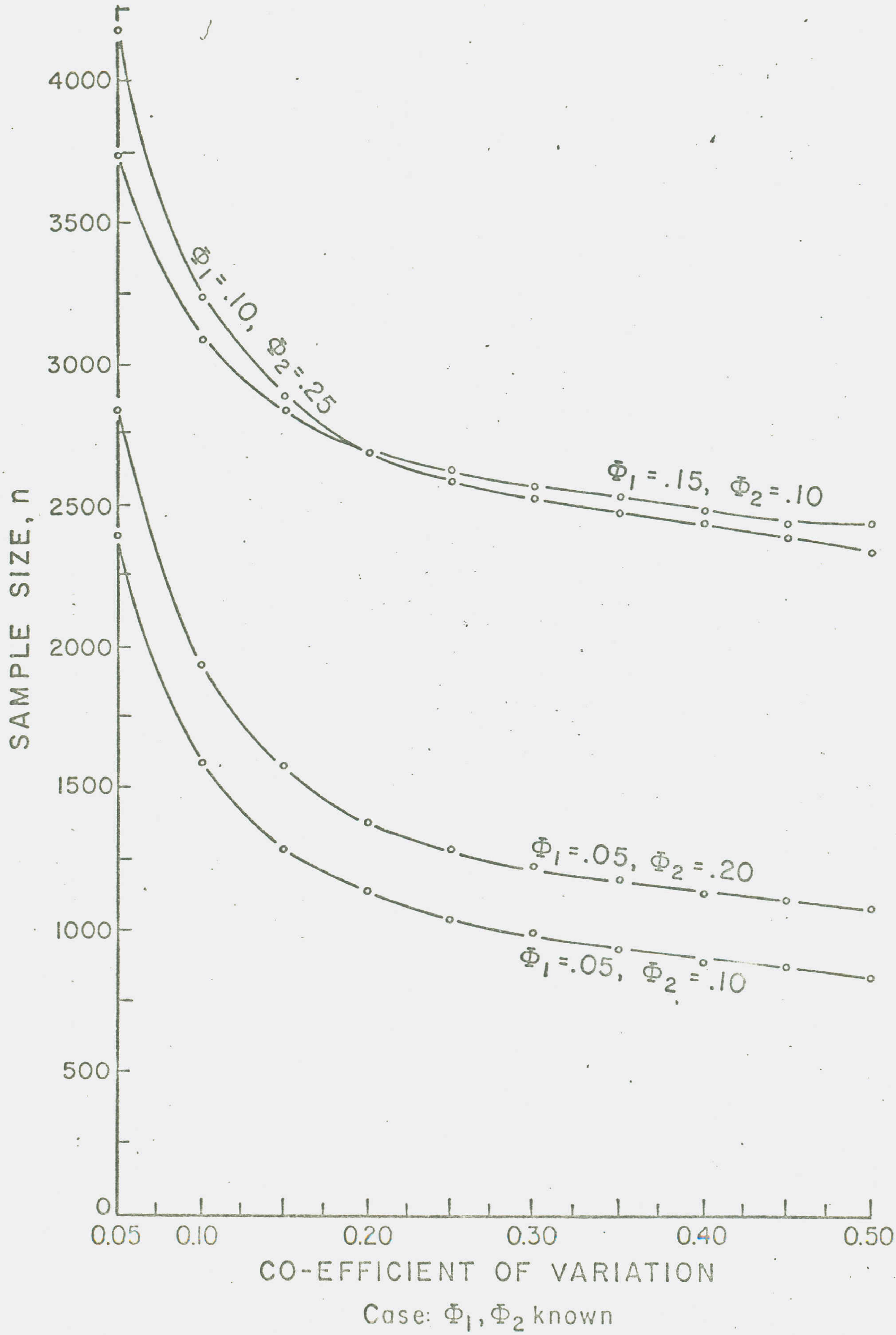
Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 20$

FIGURE 6



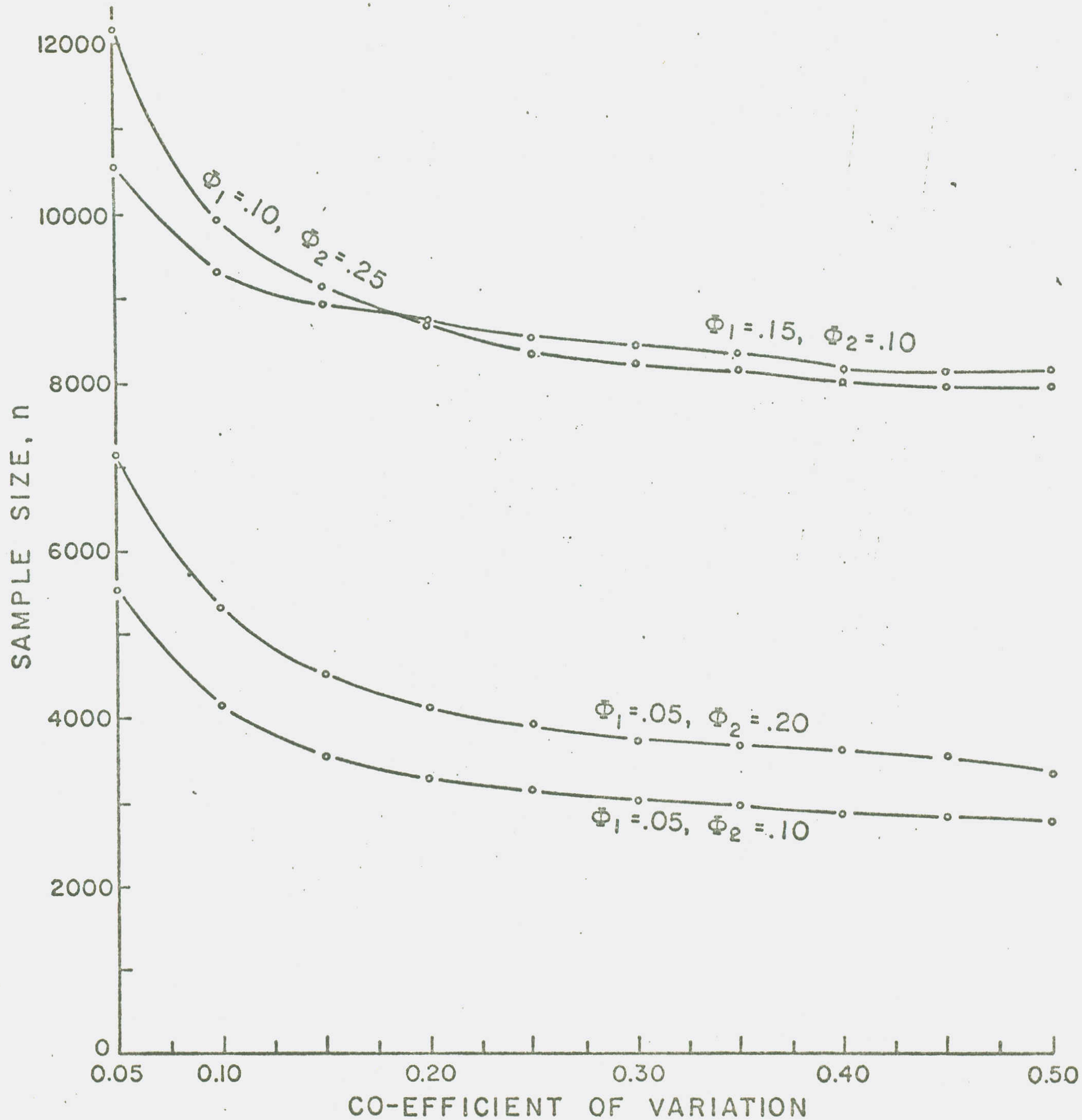
Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 20$

FIGURE 7



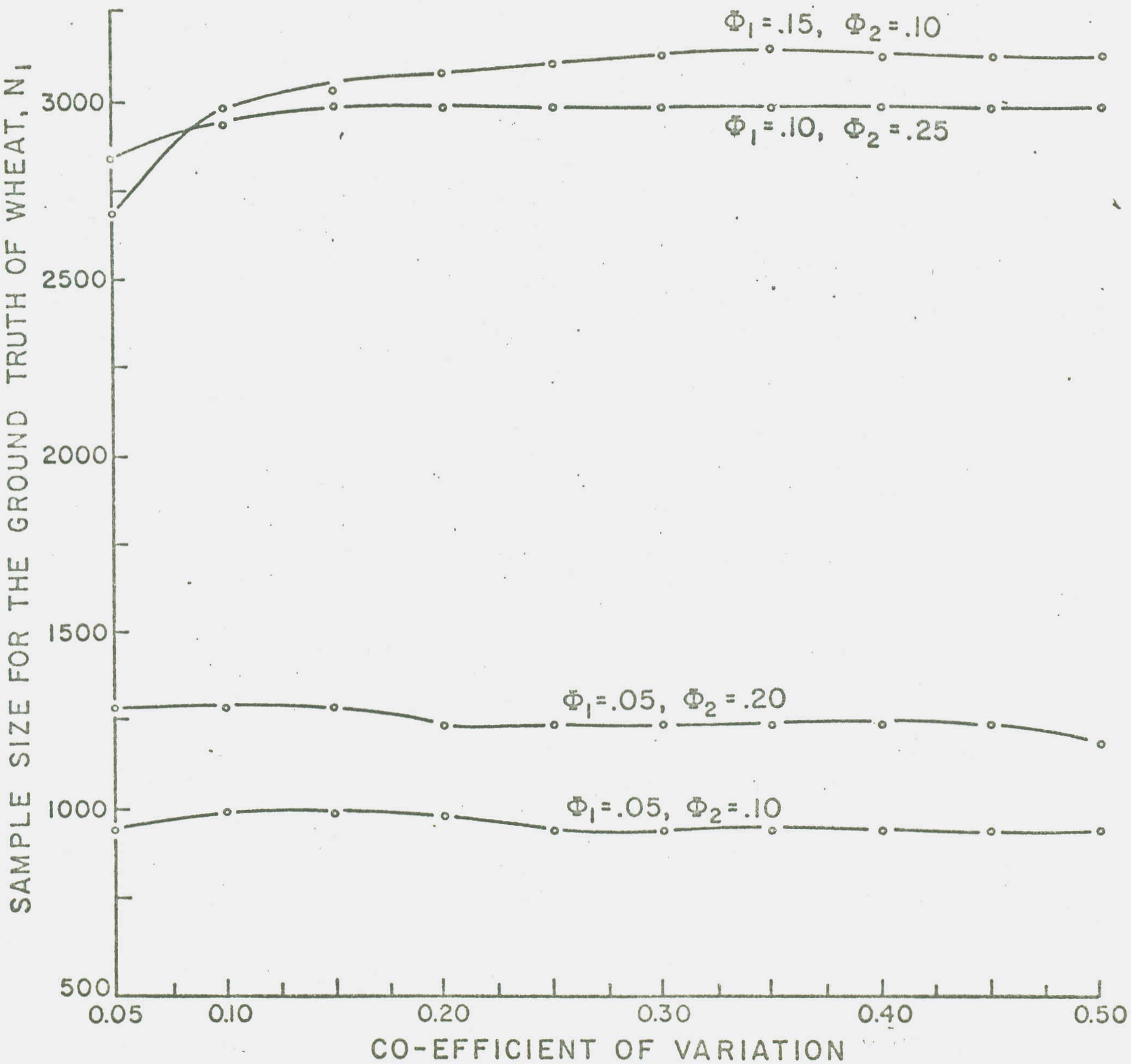
4000
3500
3000
2500
2000
1500
1000
500
0
SAMPLE SIZE, n
$\Phi_1 = .10$, $\Phi_2 = .25$
$\Phi_1 = .15$, $\Phi_2 = .10$
$\Phi_1 = .05$, $\Phi_2 = .20$
$\Phi_1 = .05$, $\Phi_2 = .10$
0.05
0.10
0.20
0.30
0.40
0.50
CO-EFFICIENT OF VARIATION
Case: Φ_1, Φ_2 known

FIGURE 8



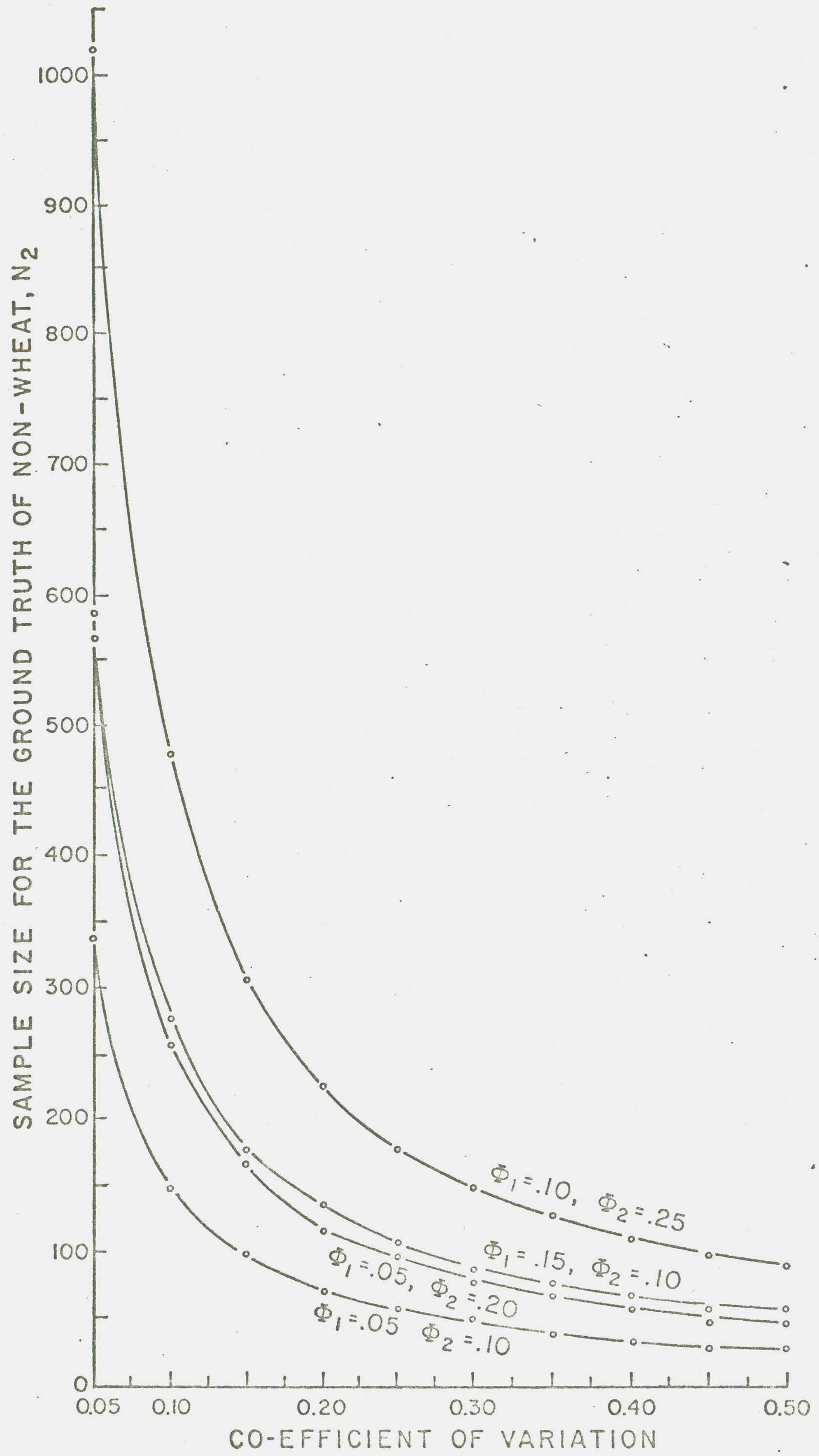
Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 5$

FIGURE 9



Case: Φ_1, Φ_2 unknown and $c_2/c_1=5$

FIGURE 10



Case: Φ_1, Φ_2 unknown and $c_2/c_1 = 5$

FIGURE II

0.950	0.050	0.200	0.7625	33	105	2	39	168	1	33
0.950	0.050	0.100	0.8575	17	52	1	18	83	1	15
0.950	0.000	0.050	0.9025	10	27	0	8	41	0	7